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LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF THIN, ORTHOTROPIC RECTANGULAR PLATES WITH RECTANGULAR CUTOUTS WITH FIXED BOUNDARIES

R. E. Rossi

Department of Engineering, *Universidad Nacional del Sur*, 8000 *Bahía Blanca*, *Argentina*

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1. INTRODUCTION

The problem of transverse vibrations of thin, isotropic and orthotropic rectangular plates with rectangular or circular holes when the edge of the perforation remains free has attracted considerable interest, the problem being motivated by operational reasons (passage of ducts or conduits, electrical cables, etc.) [1–4]. Approximate, analytical solutions have been employed and the results compared with finite element (FE) determinations.

Considerably less work has been performed when the edges of the cutout are fixed, e.g., simply supported or clamped. An exception to this is a rather recent publication where membranes and isotropic plates have been analyzed [5].

The present study deals with the determination of the fundamental frequency of transverse vibration of the structural system shown in Figure 1. Orthotropic constitutive relations are assumed. All the possible combinations of boundary supports for the outer and inner boundaries have been considered and are summarized in Table 1.

The geometric configurations correspond to plates of outer boundaries and concentric inner holes of equal aspect ratio. Hence, $a/b = c/d$; see Figure 1. The case where $c/a = 0$ corresponds to a central, concentrated support. Approximate

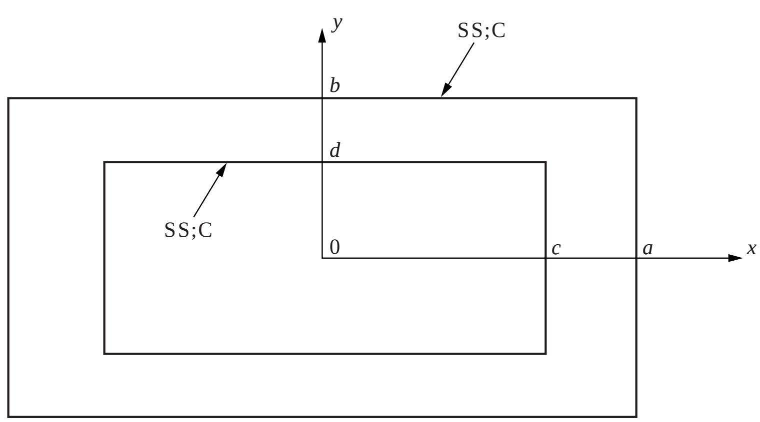

Figure 1. Orthotropic doubly connected plate executing transverse vibrations ($a/b = c/d$).

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TABLE 1

Combinations of boundary supports

Case	Inner boundary	Outer boundary
(A)	SS	SS
(B)	C	SS
(C)	SS	C
(D)	C	C

analytical solutions appear, at best, exceedingly difficult. In view of this, a numerical solution is provided using the finite element method and employing a well known code [6]. On the other hand and in order to ascertain the relative accuracy of the results, one configuration was also solved using the orthotropic finite element developed in reference [7] which is based on the very accurate element due to Bogner *et al*. [8] for isotropic plates. In view of the symmetry of the configuration and of the fundamental mode shape, one-quarter of the doubly connected plate was modelled for this particular case.

2. NUMERICAL RESULTS

All calculations have been performed for a hypothetical orthotropic structural element for which $D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$; $\nu_2 = 0{\cdot}3$ where Lekhnitskii's classical notation has been used [9]. On the other hand all the geometric configurations are defined by the equality $a/b = c/d$, as previously stated.

Table 2 depicts a comparison of values of the fundamental frequency coefficient ($\Omega_1 = \omega_1(2a)^2\sqrt{\rho h/D_1}$, where ρ is the mass density of the material and h is the thickness of the plate) for the case $a/b = 3$; $c/a = 0{\cdot}8$ obtained for different finite element algorithmic procedures [6–8], for the combinations of boundary conditions defined previously as (A), (B), (C) and (D) in Table 1. The agreement is excellent from a practical viewpoint.

Table 3 presents fundamental eigenvalues of the configurations shown in Figure 1 as a function of a/b and c/a. No claim of originality is made but since

TABLE 2

Comparison of fundamental frequency coefficients in the case of the configuration shown in Figure 2, *as a function of the number of elements*, *using different FE codes and for different combinations of boundary conditions*

FE code	Number of elements	SS–SS	C–SS	SS–C	C–C
[7–8]	243	1088·0	1605·0	1628·2	2294·1
[7–8]	432	1087·2	1604·9	1627·6	2294·0
[6]	2700	1085·5	1604·9	1626·6	2294·1
[6]	10800	1084·8	1604·8	1626·0	2294·0

TABLE 3

Fundamental eigenvalues $\omega_1(2a)^2\sqrt{\rho h/D_1}$ *of the configuration shown in Figure* 1 *as a function of* a/b *and* c/a

a/b	c/a	SS–SS	C–SS	SS–C	C–C	Number of elements
3	0	110·84	115·37	183·13	188·05	7500
	0·2	151·80	155·31	288·44	233·49	7200
	0·4	209·68	231·59	298·08	330·52	6300
	0·6	351·32	447·20	494·82	628·37	4800
	0·8	1085·8	1604·9	1626·8	2294·2	6912
3/2	0	59·17	63·81	86·18	92·24	5400
	0·2	96·65	101·39	139·20	147·08	5184
	0·4	148·99	177·66	212·48	256·44	4536
	0·6	282·00	393·60	416·78	568·24	3456
	0·8	1015·3	1553·4	1564·5	2248·1	1944
1	0	40·38	43·59	58·71	62·92	6400
	0·2	66·01	69·30	95·77	101·11	6144
	0·4	103·20	123·05	147·84	178·49	5376
	0·6	197·16	275·42	292·58	399·04	4096
	0·8	715·71	1095·2	1104·5	1587·2	3600
2/3	0	24·21	25·79	36·67	38·61	5400
	0·2	36·98	38·32	53·03	55·17	5184
	0·4	54·09	62·34	75·82	88·37	4536
	0·6	96·77	130·17	139·26	185·06	3456
	0·8	326·87	494·19	498·24	711·54	1944
1/3	0	13·81	14·14	25·29	25·58	7500
	0·2	16·99	17·19	28·22	28·46	7200
	0·4	21·85	23·18	33·22	35·02	6300
	0·6	33·71	40·14	47·80	56·67	4800
	0·8	92·17	131·15	134·88	185·52	6192

Note: the case $c/a = 0$ corresponds to a concentric, point support.

the problem is of considerable interest in civil and naval engineering structures, it is hoped that the present approach and results will be of interest to structural designers.

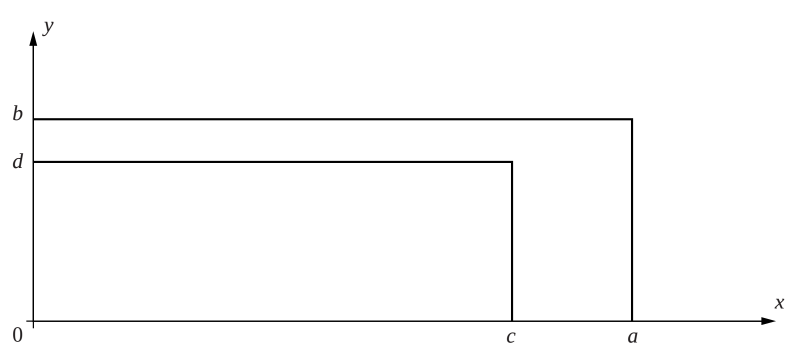

Figure 2. One-quarter of the configuration corresponding to $a/b = 3$; $c/a = 0{\cdot}8$.

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